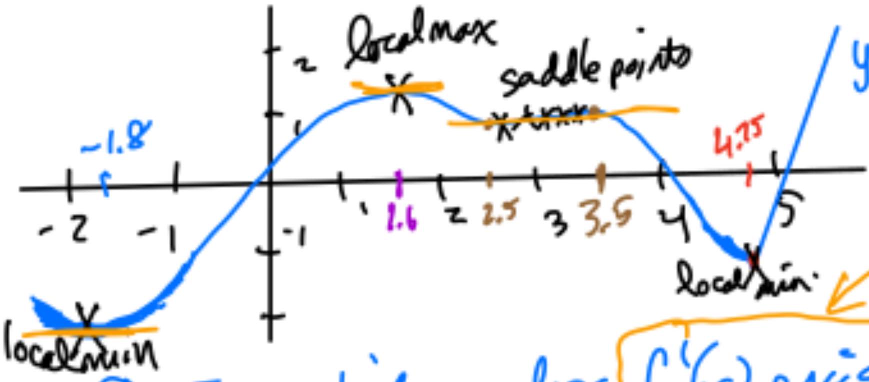
Question: Let $y=f(x)$ be graphed below.



function is differentiable at x $\Leftrightarrow$ has tangent line @ x. $\Leftrightarrow$ looks like a straight line if we zoom in at x.

ⓐ For which x does $\boxed{f'(x) \text{ exist}}$?

$x \in (-2.3, 4.75) \cup (4.75, 5.5)$ (ie. all pts except $x=4.75$)

ⓑ For which x is $\boxed{f'(x) > 0}$?

x where slope is positive
tangent line

$x \in (-1.8, 1.6) \cup (4.75, 5.5)$

f is strictly increasing at these values of x

ⓒ For which x is $f'(x) < 0$?

$x \in (-2.2, -1.8) \cup (1.6, 2.5) \cup (3.5, 4.75)$

f is strictly decreasing here

ⓓ For which x is $f'(x) = 0$?

$x = -1.8$ or $x = 1.6$ or $x \in [2.5, 3.5]$

horizontal tangent at these places.

ⓔ The x-values of all the points marked with an × are called critical points.

— where $f'(x) = 0$ or $f'(x)$ does not exist.

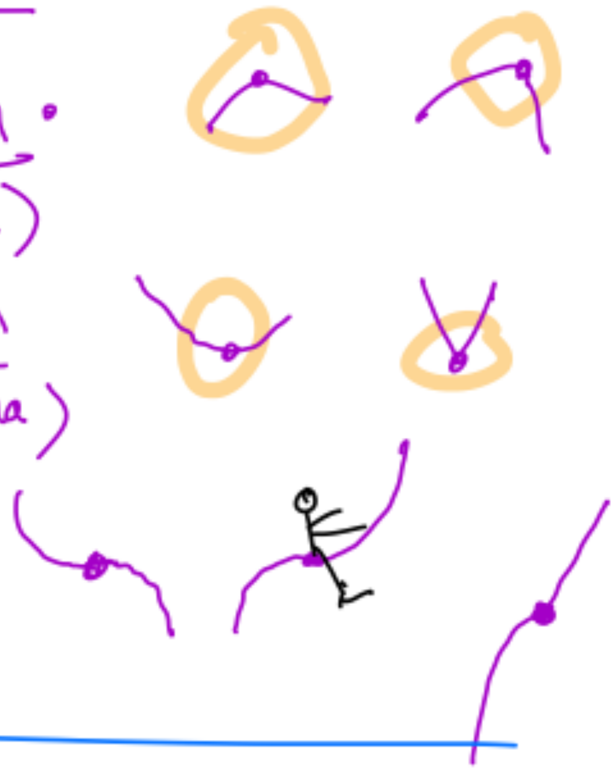
horizontal tangents.   corners

Kinds of critical points:

local maximum.
(plural maxima)

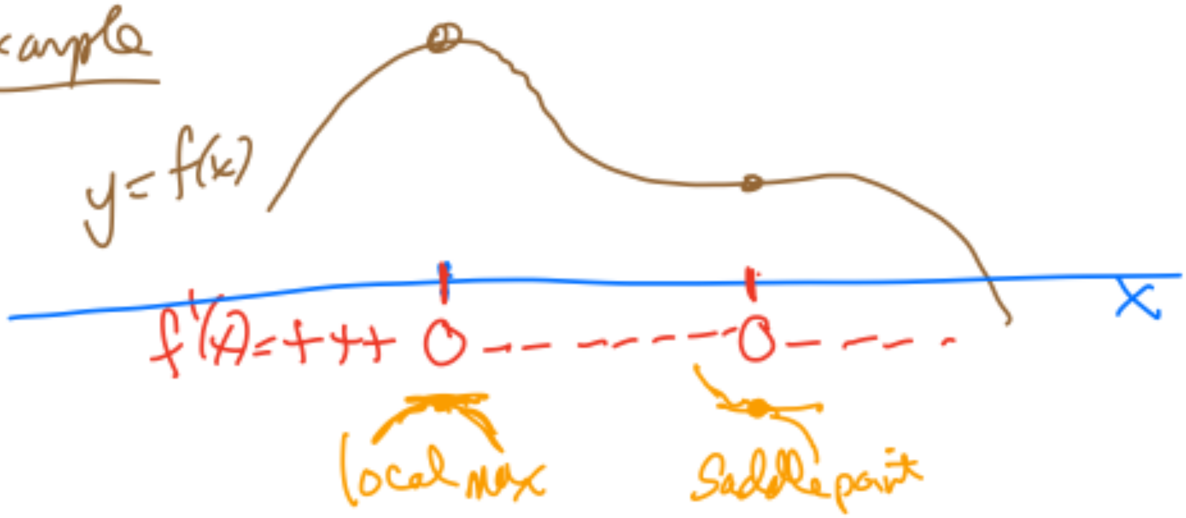
local minimum
(plural minima)

saddle point



Important Fact: We can use the values (0, +, −) of the derivative to determine where a function has critical points & what kind they are.

example



Example Let $g(x) = x^3 - x^2$.

Find all the critical points of g and classify them.

← local min, max or saddle.

$g'(x) = 3x^2 - 2x = 0$ or DNE for x critical

$g'(x) = x(3x-2) = 0$

$\Rightarrow x = 0$ or $x = \frac{2}{3}$

two critical point.

Let's look at sign of $g'(x)$

local max   local min

$\Rightarrow g(x)$

$g'(x) = + + + 0 - - - 0 + + + +$

$-1 \quad 0 \quad 1/2 \quad 2/3 \quad 1$

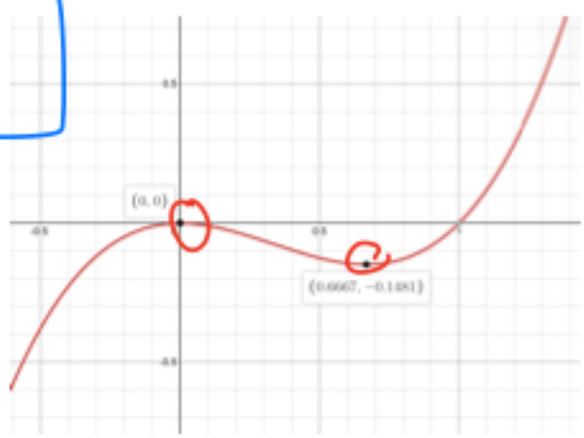
$g'(x) = (-1)(3(-1)-2)$
$= (-1)(-5) = 5$

$\frac{1}{2}\left(3\left(\frac{1}{2}\right) - 2\right)$
$= \frac{1}{2}\left(\frac{3}{2} - 2\right) = \frac{1}{2}\left(-\frac{1}{2}\right) = -$

$x = 0$ is a local max
& $x = \frac{2}{3}$ is a local min

check:

$y = g(x)$ graph ↗



Note: The word "relative max" is the same as local max. (same for "relative min" & local min)

$\boxed{\text{Example}}$ Find & classify the critical points of $y = xe^{-x^2/2}$.

Solution: Take the derivative

$$y' = e^{-x^2/2} + x \cdot e^{-x^2/2} \cdot \left(\frac{-x^2}{2}\right)'$$

$-\frac{1}{2}(2x) = -x$

$$y' = e^{-x^2/2} - x^2 e^{-x^2/2}.$$

Critical pts

$$e^{-x^2/2} - x^2 e^{-x^2/2} = 0$$

$$(1 - x^2)e^{-x^2/2} = 0$$

$$(1+x)(1-x)e^{-x^2/2} = 0 \iff x = 1 \text{ or } x = -1 \text{ or } \boxed{e^{-x^2/2} = 0} \text{ impossible}$$

$\Rightarrow y =$ 

$y' =$ − − − 0 + + + + + + + + 0 − − − − − − −

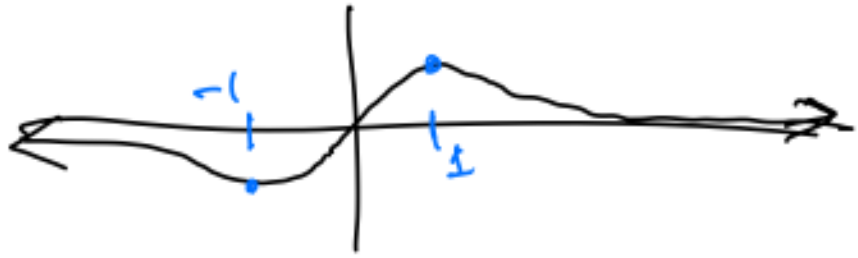
−1   1 x

$y'(-2) = (-1)\cdot(3)e^{\sim}$
  −   +   +

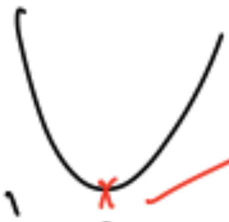
$(1)(+1)\,1 = 1$

$\Rightarrow$ $x = -1$ is a local min
  $x = 1$ is a local max

$y = xe^{-x^2/2}$



$\boxed{\text{Example}}$ Where is the vertex of the parabola $y = 14x^2 - 22x + 749$?



$y' = - - 0 + + +$

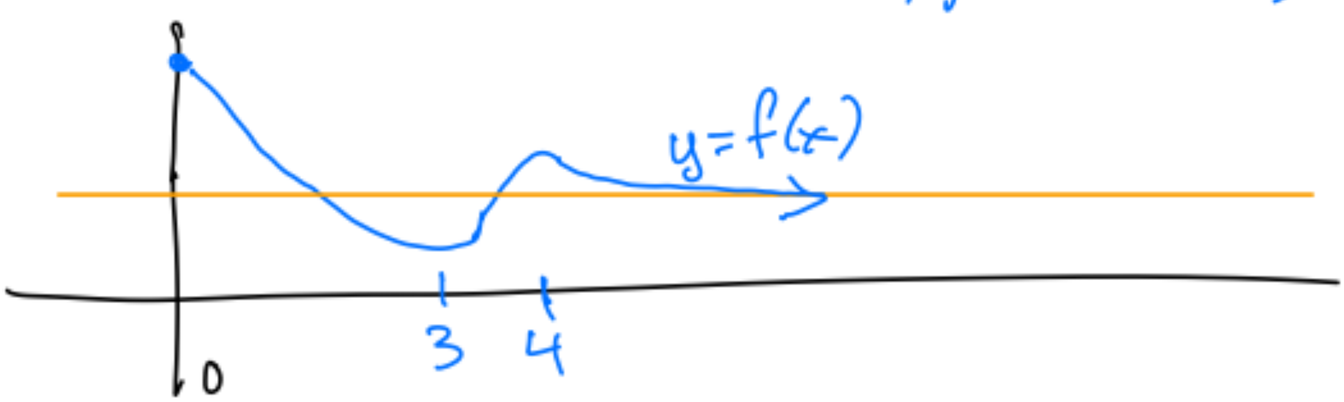
$y' = 0 = 28x - 22$

$\Rightarrow x = \frac{22}{28} = \frac{11}{14}.$

$y = 14\left(\frac{11}{14}\right)^2 - 22\left(\frac{11}{14}\right) + 749 = \text{Bubba}$

vertex: $\boxed{(x, y) = \left(\frac{11}{14}, \text{Bubba}\right)}$

Sometimes we want to find the absolute maximum or minimum of a function on an interval. (also called the global maximum /global minimum).



In this picture, $x=0$ is the absolute maximum of f on the interval $[0,\infty)$.

$x=3$ is a local min that is also the absolute minimum.

$x=4$ is a local max that is not an absolute max.

Facts about absolute mins & maxes.

- Given a differentiable function on an interval, ~~if~~ it has an absolute maximum, then that value occurs either when $x=$ a critical point or $x=$ endpoint of the interval.

  (same for absolute minimum.)

- If you know a fcn has abs minimum, you can find it by:
  - Find all critical pts & end points
  - plug those #s into the original fns ← pick the point with lowest value.

  (similarly for abs maximum).

- (Extreme value theorem) A differentiable function on a closed interval $[a,b]$ always has an absolute maximum and an absolute minimum.

- For the closed interval, you will always find the abs min & max by finding all cr. points & endpoints & plugging into the function.

Quiz

1. What is your name, spelled backwards?
2. Draw a fish.
3. Simplify $\dfrac{x+2-3x^2}{x-1}$.
4. Find the derivative: (Derivadorize :)

(a) $\cos(3x)$

(b) e^{3x}

(c) 2^x

(d) $\arctan(x)$

(e) $\ln(x)$

(f) $\sqrt{x}\,\tan(x)$.